SIMILAR SOLUTIONS OF THE MAGNETOHYDRODYNAMIC EQUATIONS WITH FINITE GAS CONDUCTIVITY

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We examine the one-dimensional nonstationary motion of a conducting gas in an external magnetic field oriented normal to the moving medium. It is assumed that the gas is ideal, and that viscosity and heat conductivity are absent. Then, in magnetohydrodynamic approximation, the system of equations describing the motion of a conducting gas in a magnetic field has the form

$$\frac{\partial H}{\partial t}+\frac{1}{r^{\alpha}}\frac{\partial}{\partial r}(r^{\alpha}vH)=\frac{1}{4\pi r^{\alpha}}\frac{\partial}{\partial r}\left(\frac{r^{\alpha}}{\sigma}\frac{\partial H}{\partial r}\right)$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial}{\partial r}\left(P+\frac{H^2}{8\pi}\right),\qquad P=R\rho T$$

$$\frac{\partial \rho}{\partial t}=-\frac{1}{r^{\alpha}}\frac{\partial}{\partial r}(r^{\alpha}v\rho)$$

$$c_v\rho\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial r}\right)=-\frac{P}{r^{\alpha}}\frac{\partial}{\partial r}(r^{\alpha}v)+\frac{1}{16\pi^2\sigma}\left(\frac{\partial H}{\partial r}\right)^2 \tag{0.1}$$

The denotations, here, are the generally accepted ones. In the plane-symmetrical case $\alpha=0$, while in the case of cylindrical symmetry, $\alpha=1$.

Furthermore, we assume that the conductivity of the gas depends only on temperature and is defined by the relation

$$\sigma=CT^n \quad (n\geqslant 0) \tag{0.2}$$

If the motion of the gas is accompanied by the development of a shock wave, then the conditions at the shock wave front must be added to the system (0.1). Assuming that the shock wave is a gas dynamic one, we write

$$\rho_1(v_1-D)=\rho_2(v_2-D),\quad P_1+\rho_1(v-D)^2=P_2+\rho_2(v_2-D)^2$$

$$\frac{\varkappa}{\varkappa-1}\frac{P_1}{\rho_1}+\frac{(v_1-D)^2}{2}=\frac{\varkappa}{\varkappa-1}\frac{P_2}{\rho_2}+\frac{(v_2-D)^2}{2},\quad H_1=H_2 \tag{0.3}$$

Subscript 1 refers to physical quantities in front of the shock wave, subscript 2 to those behind the shock wave, D is the velocity of the shock wave, and $\varkappa$ the ratio of specific heats.

We determine a transformation which does not change the form of the Eqs. (0.1) and (0.3). Let

$$r=\varepsilon_1 r,\ t=\varepsilon_2 t,\ H=\varepsilon_3 H,\ v=\varepsilon_4 v,\ P=\varepsilon_5 P,\ \rho=\varepsilon_6\rho \tag{0.4}$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$ are certain constant coefficients which define the transformation of the corresponding variables for which the form of the Eqs. (0.1), (0.3) does not change.

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By substituting (0.4) into (0.1), we get the following two-parameter group of transformations, which holds also for the relations (0.3):

$$r = \varepsilon_1 r,\quad t = \varepsilon_1^{\frac{2(n+1)}{2n+1}} t,\quad H = \varepsilon_1^{m} H,\quad v = \varepsilon_1^{\frac{-1}{2n+1}} v\quad P = \varepsilon_1^{2m} P,\quad \rho = \varepsilon_1^{2\left(m+\frac{1}{2n+1}\right)} \rho,\quad T = \varepsilon_1^{\frac{-2}{2n+1}} T$$

According to [1], similarity solutions of system (0.1) will have the form

$$H = H_0 r^{m} h(\xi),\quad v = v_0 r^{\frac{-1}{2n+1}} u(\xi),\quad T = T_0 r^{\frac{-2}{2n+1}} \theta(\xi),\quad \rho = \rho_0 r^{2m+\frac{2}{2n+1}} \chi(\xi) \tag{0.5}$$

The similarity variable is

$$\xi = C_0 r / t^{\frac{2n+1}{2(n+1)}},\quad C_0 = C^{\frac{1}{2(n+1)}}\, c_v^{\frac{-n}{2(n+1)}} \tag{0.6}$$

Here, H_0, v_0, ρ_0, and T_0 are certain dimensional constants.

The dimensionless functions $h(\xi)$, $u(\xi)$, and so on, are defined by a system of ordinary differential equations which derive from (0.1) with consideration of (0.5)

$$-\frac{1}{N} h' + \xi^{-(m+1+\alpha)} (\xi^{\alpha+m-N_1} uh)' - \frac{\xi^{-(m+1+\alpha)}}{A} \left[\frac{\xi^{\alpha+2nN_1}}{\theta^n} (\xi^m h)'\right]' = 0$$

$$-\frac{1}{N} u' + \frac{u}{\xi} (\xi^{-N_1} u)' + \frac{\xi^{-(2m+N)}}{\chi} [\xi^{2m} (\chi\theta + Bh^2)]' = 0 \qquad -\frac{1}{N} \chi' + \xi^{-(\alpha+2m+1+2N_1)} (\xi^{\alpha+2m+N_1} u\chi)] = 0$$

$$-\frac{1}{N} \theta' + \xi^{-2nN_1} u (\xi^{-2N_1}\theta) + (\varkappa - 1) \xi^{-(\alpha+1)} \theta (\xi^{\alpha-N_1} u)' - K \frac{\xi^{-(2m+N_1)}}{\theta^n \chi} (\xi^m h)'^2 = 0 \tag{0.7}$$

Here, the following notations are introduced for the dimensionless parameters:

$$A = 4\pi C T_0^n v_0,\quad B = H_0^2/8\pi P_0,\quad K = \frac{2B(\varkappa-1)}{A},\quad N = \frac{2(n+1)}{2n+1},\quad N_1 = \frac{1}{2n+1}$$

Furthermore, $T_0 = C_0^{-2N}/R$, $v_0 = C_0^{-N}$, $\rho_0 = P_0 C_0^{2N}$. The prime denotes differentiation with respect to ξ.

In this way, the solution of the system of partial differential equations (0.1) reduces to that of a nonlinear system of ordinary fifth-order differential equations. For individual values of the parameter m, we present some particular solutions of system (0.7) which have the form

$$h = C_1 \xi^{\nu},\quad u = C_2\, \xi^{\mu},\quad \chi = C_3 \xi^{\delta},\quad \theta = C_4 \xi^{\varepsilon} \tag{0.8}$$

For $m = -N(1+\alpha)$, $\varkappa = 2$

$$\nu = (1+\alpha) N,\quad \mu = N,\quad \delta = \frac{N}{1-N}[(1-2N)\alpha - 1],\quad \varepsilon = \frac{N}{1-N}(\alpha + 3 - 2N)$$

$C_2=1$, while C_1, C_2, and C_4 are arbitrary constants. By substituting (0.8), with consideration of (0.6) and the latter relations, into (0.5), we get for the physical quantities

$$H = H_0 \frac{C_1}{t^{1+\alpha}},\quad \rho = \rho_0 r^{2N_1 - \frac{N(3+2N)}{1-N}}\, t^{-\alpha + \frac{N\alpha+1}{N-1}}$$

$$T = T_0 C_4 r^{-2N_1 + \frac{N(\alpha-3-2N)}{1-N}}\, t^{\frac{\alpha-3-2N}{N-1}},\qquad v = \frac{r}{t} \tag{0.9}$$

For $m = -2N(1+\alpha)/[2+(1+\alpha)(\varkappa-1)]$

$$\nu = -m,\quad \mu = N,\quad \delta = 2\frac{\alpha + 2m + N_1 + N}{(1+\alpha)(\varkappa-1)},\quad \varepsilon = 2N$$

$$C_2 = \frac{2}{2+(1+\alpha)(\varkappa-1)},\quad C_4 = \frac{C_2(1-C_2)}{2m+\delta+2N}$$

C_1 and C_2 are arbitrary constants.

$$H = H_0 C_1 t^{-\frac{2(\alpha+1)}{2+(\alpha+1)(\varkappa-1)}}, \quad \rho = \rho_0 C_3 \frac{r^{2(m+N_1)+\gamma}}{t^{\gamma}}$$

$$T = T_0 C_4 \frac{r^2}{t^2}, \quad v = \frac{r}{t} \qquad \left(\gamma = \frac{\alpha + 2m + N_1 + N}{(1+\alpha)(\varkappa - 1)}\right) \tag{0.10}$$

For $-{}^1/_3(1+4N_1)$, $\alpha = 0$

$$\nu = {}^4/_3 N, \quad \mu = N, \quad \delta = {}^2/_3 N, \quad \varepsilon = 2N_1, \quad C_2 = {}^2/_3$$

$$C_1 = \frac{1}{3}\left(\frac{C_3}{B}\right)^{1/2}, \quad C_4 = \left[\frac{K}{6B}\frac{N}{3N_1 + (\varkappa - 1)N}\right]^{1/(n+1)}$$

Here only C_3 is an arbitrary constant.

$$H = H_0 C_1 r t^{-4/3}, \quad \rho = \rho_0 C_3 t^{-2/3}, \quad T = T_0 C_4 t^{-2N_1/N}, \quad v = \frac{r}{t} \tag{0.11}$$

It should be noted that in solutions of the form (0.9) and (0.10), the magnetic field is independent of the coordinate, and is a function only of time.

1. Some Integrals of System (0.7). In the case of

$$m = -\frac{1 + 2N_1 + \alpha}{2} \tag{1.1}$$

the order of the system (0.7) can be lowered by unity by integrating the third equation and substituting the expression obtained for

$$\chi = \frac{C_1 \xi^N}{N^{-1}\xi^N - u} \tag{1.2}$$

into the remaining three equations (it is assumed that $u \neq N^{-1}\xi^N$).

If, furthermore, $N_1 = {}^1/_2(1+\alpha)$ $(n = (1-\alpha)/2(1+\alpha))$ is assumed, then the first equation in (0.7) is singly integrated simultaneously, after which we obtain a system of three first-order differential equations

$$-\beta h' + \xi^{-\frac{\alpha+3}{2}} uh - \frac{1}{A\theta^n} \xi^{\frac{1+\alpha}{2}} (\xi^{-(1+\alpha)} h)' = C$$

$$-\beta u' + \frac{u}{\xi}\left(\xi^{-\frac{1+\alpha}{2}} u\right)' + \frac{1}{\chi} \xi^{\frac{3\alpha+1}{2}} [\xi^{-2(1+\alpha)}(\chi\theta + Bh^2)]' = 0$$

$$-\beta\theta' + \xi^{-\frac{1-\alpha}{2}} u (\xi^{-1-\alpha}\theta)' + (\varkappa - 1)\xi^{-1-\alpha}\theta\left(\xi^{-\frac{1-\alpha}{2}} u\right)' - \frac{K}{\theta^n \xi} \xi^{\frac{3(1+\alpha)}{2}} \left(\frac{h}{\xi^{1+\alpha}}\right)'^2 = 0 \tag{1.3}$$

Here, $\beta = 2/3 + \alpha$, while $\chi(\xi)$ is defined by formula (1.2) for $N = 3 + \alpha/2$.

For a small hydromagnetic interaction parameter

$$BA \ll 1 \tag{1.4}$$

where the influence of the electromagnetic field on the motion of the medium may be neglected, Eq. (1.3) is integrable in final form if $\varkappa = 2$, $\alpha = 1$ is postulated.

Indeed, by omitting the last term (the assumption (1.4)) in the third equation of system (1.3), and setting $\varkappa = 2$, we get

$$\theta(\xi) = C_2 \frac{\xi^{\frac{3+\alpha}{2}}}{-\beta\xi^{\frac{3+\alpha}{2}} + u} \tag{1.5}$$

Here, α may either vanish or be equal to unity. From the second equation in (1.3), for $\alpha = 1$, with allowance for (1.2), (1.4), (1.5), we get

$$u^3 - 3\beta\xi^2 u^2 + 2(\beta^2\xi^2 - C_3)\xi^2 u + 2\xi^2(2C_2 + C_3\beta\xi^2) = 0$$

Now, from the first equation in (1.3), we determine $h(\xi)$ ($\alpha = 1$, $n = 0$)

$$h(\xi) = \xi^2 \exp\left[-\frac{\beta R_m}{2}\xi^2 + \beta R_m \int \frac{u d\xi}{\xi}\right]\left\{-C_4 R_m \int \exp\left[\frac{\beta R_m}{2}\xi^2 - \beta R_m \int \frac{u d\xi}{\xi}\right]\frac{d\xi}{\xi} + C_5\right\}$$

The arbitrary constants C_1, C_2, C_3, C_4, and C_5 are determined from the boundary conditions.

It is noteworthy that to the condition (1.1), for which the integral (1.2) exists, there corresponds the case of constant mass in the region of motion Ω studied, i.e.,

$$\int_\Omega \rho d\Omega = \text{const}$$

2. Motion of a Conducting Gas in a Magnetic Field under the Action of a Piston. Assume that at a moment of time $t = 0$, a compressible gas occupies a half-space $r > 0$, and that it is bounded by a plane ($\alpha = 0$) and by a cylindrical ($\alpha = 1$) infinitely conducting piston which starts to move along the axis r at a moment of time $t = 0$. In this case, a shock wave, which disturbs the initial distribution of the physical quantities, will propagate in the gas.

Let us examine the case $m = 0$, $n \geq 0$. Then the similar solution of system (0.1) will take the form

$$H = H_0 h(\xi), \quad v = v_1 t^{-1/2(n+1)} u_1(\xi), \quad P = P_0 G(\xi)$$
$$T = T_1 t^{-1/(n+1)} \theta_1(\xi), \quad \rho = \rho_1 t^{1/(n+1)} \chi_1(\xi)$$
$$u_1 = \xi^{-1/(2n+1)} u(\xi), \quad \theta_1 = \xi^{-2/(2n+1)} \theta(\xi), \quad \chi_1 = \xi^{2/(2n+1)} \chi(\xi)$$

Here, H_0, v_1, P_0, T_1, and ρ_1 are the corresponding dimensional constants.

At the initial moment of time, the distribution of the physical quantities over the r coordinate was as follows

$$H = H_0, \quad v = 0, \quad T = T_0 r^{-2/(2n+1)}, \quad \rho = \rho_0 r^{2/(2n+1)}$$

Let ξ_s and ξ_0 denote the positions of the piston and the shock wave, respectively, in the space of the similarity variable ξ.

From the similarity condition of the problem, we obtain the law that governs the motion of the piston.

$$r_s = \frac{\xi_s}{C_0} t^{\frac{2n+1}{2(n+1)}}$$

(here, and in the following, subscript s denotes the conditions on the piston). The speed of piston motion is

$$v_s = \frac{1}{N}\frac{\xi_s}{C_0} t^{-1/2(n+1)}$$

Let us go over to the variable $\zeta = \xi/\xi_0$, and write the system (0.7), for $m = 0$, in the form

$$u' = \frac{\zeta^N F}{\zeta^{2N} F^2 - \varkappa\theta}\left[N_1 \frac{u^2}{\zeta} + \frac{\varkappa(\alpha - N_1)}{\zeta^{N+1}}\frac{\theta u}{F} - \frac{K h'^2}{\zeta^{N-1}\theta^n \chi F} - 2B\frac{hh'}{\chi}\right]$$

$$\chi' = -\frac{\chi}{\zeta^N F}\left[u' + (\alpha + N_1)\frac{u}{\zeta}\right]$$

$$\theta' = \frac{\theta}{\zeta^{N+1}F}\{[2N_1 - (\varkappa - 1)(\alpha - N_1)]u - (\varkappa - 1)\zeta u'\} + \frac{Kh'^2}{\zeta^{N-1}\theta^n \chi F}$$

$$h'' = A\theta^n\left\{\left[-\beta_1\zeta^{N_1} + \frac{u}{\zeta} + \frac{n}{A}\frac{\theta'}{\theta^{n+1}} - \frac{\alpha + 2nN_1}{A}\frac{1}{\zeta\theta^n}\right]h' + \left(\frac{u'}{\zeta} + \frac{\alpha - N_1}{\zeta^2}u\right)h\right\}$$

$$F = -\beta_1 + \zeta^{-N}u, \quad \beta_1 = \frac{\xi_0{}^N}{N}$$

(2.1)

The boundary conditions derive from the relations (0.3) at the shock wave

$$u = \frac{2}{\varkappa + 1}\beta_1\left(1 - \frac{\varkappa}{\beta_1}\right), \quad \chi = \left[\frac{\varkappa - 1}{\varkappa + 1} + \frac{2\varkappa}{\beta_1^2(\varkappa + 1)}\right]^{-1}$$

$$\theta = \beta_1^2\left(2 - \frac{\varkappa + 1}{\beta_1^2}\right)\frac{\varkappa - 1 + 2\varkappa/\beta_1^2}{(\varkappa + 1)^2}, \quad h = 1 \quad \text{for} \quad \zeta = 1$$

(2.2)

while the conditions on the piston are $u_s = \beta_1\zeta_s{}^N, h' = 0$ for $\zeta = \zeta_s$. The last condition in (2.2), $h'(\zeta_s) = 0$, signifies the absence of diffusion of the magnetic field into an infinitely conducting piston. By changing the variables

$$u = \beta\zeta^N w(\zeta), \qquad \chi = \chi(\zeta), \qquad \theta = \beta^2\zeta^{2N}z(\zeta) \tag{2.3}$$

Eq. (2.1) is reduced to the form

$$\frac{d\zeta}{dw} = \zeta\frac{F_1^2 - \varkappa z}{Nw(\varkappa z - F_1^2) + N_1 w^2 F_1 - \Phi + \varkappa(\alpha - N_1)wz\Phi} = \varphi$$

$$\Phi = K_1\zeta^{-(2n+3)N}\frac{y^2}{\chi z^n} + \frac{2B_1 Fhy}{\zeta^{2N}\chi}$$

$$\frac{d\chi}{dw} = -\frac{\chi}{F_1}[1 + (N + N_1 + \alpha)w\varphi], \qquad \frac{dh}{dw} = y\varphi$$

$$\frac{dz}{dw} = z\left\{\frac{1 - \varkappa}{F_1} + \left[-2N + \frac{2N_1 - (\varkappa - 1)(1 + \alpha)}{F_1}w + \frac{K_1 y^2}{\zeta^{(2n+3)N}\chi F_1 Z^{n+1}}\right]\varphi\right\}$$

$$\frac{dy}{dw} = \left\{\left[1 + A_1\zeta^{2(n+1)}z^n F_1 + n\left(2N - \frac{1}{z\varphi}\frac{dz}{dw}\right) - \alpha - 2nN_1\right]y + A_1\zeta^{2(n+1)}z^n\left[(1 + \alpha)w + \frac{1}{\varphi}\right]h\right\}\varphi$$

$$F_1 = 1 - w, \ K_1 = \beta_1^{-(2nN+3)}K, \ B_1 = \beta_1^{-2}B, \ A_1 = \beta_1^{2n+1}A \tag{2.4}$$

The boundary condition takes the form

$$\zeta = 1, \quad \chi = \left[\frac{\varkappa - 1}{\varkappa + 1} + \frac{2\varkappa}{\beta_1^2(\varkappa + 1)}\right]^{-1}, \quad z = \left[\frac{2}{\varkappa + 1} - \frac{\varkappa - 1}{\beta_1^2(\varkappa + 1)}\right]\left[\frac{\varkappa - 1}{\varkappa + 1} + \frac{2\varkappa}{\beta_1^2(\varkappa + 1)}\right]$$

$$h = 1 \quad \text{for} \quad w = \frac{2}{\varkappa + 1}\left(1 - \frac{\varkappa}{\beta_1^2}\right), \quad y = 0 \quad \text{for} \quad w = 1$$

(2.5)

The conditions on the piston, $\omega = 1$, define the quantity

$$\xi_0 = \frac{\xi_s}{\zeta_s} \tag{2.6}$$

The point $\omega = 1$ (surface of the piston) is a singular point. We seek the asymptotic expansions of the functions χ and z near $\omega = 1$ in the form

$$\chi = C_1 F_1{}^{\nu}(w), \ z = C_2 F_1{}^{\mu}(w)$$

Then, considering that for $\omega = 1$, $y \to 0$, from system (2.4) we get

$$\nu = \frac{N_1 + \alpha}{N}, \quad \mu = -\frac{(\varkappa - 1)(N_1 - \alpha) + 2N_1}{N}$$

The expansion coefficients C_1 and C_2 are determined from the condition for sewing together the numerical and asymptotic solutions.

The system (2.4) with the boundary conditions (2.5) lends itself to solution by numerical methods. Since the quantity $\beta_1 = \xi_0^N / N$ is contained explicitly in the boundary conditions (2.5), the value of the parameter ξ_0 may be assumed to be given. Then the constant ξ_s will derive from (2.6).

Let us clarify the meaning of the dimensionless parameters A and B contained in system (2.1) (or A_1, B_1 in (2.4)). To this end, we postulate that

$$R_m = 4\pi\sigma_2 v_2 r_1, \quad D_1 = \frac{H_0^2}{8\pi P_2} \tag{2.7}$$

where R_m is the magnetic Reynolds number, σ_2, v_2, and D_1 are the values of the conductivity, velocity, and the ratio of magnetic pressure to the static pressure of the gas in the shock wave, respectively, while r_1 is the distance from the piston surface to the shock wave front. Then, considering the relations (0.5), for $\xi = \xi_0$, $r = r_1$, from (2.7) we get $R_m = A\theta^n(\xi_0)\, u\,(\xi_0)$, $D_1 = B / \chi(\xi_0)\,\theta(\xi_0)$. Here, the quantities $u(\xi_0)$, $\chi(\xi_0)$, $\theta(\xi_0)$ are defined by the first three equalities in (2.2).

Numerical computations were performed on an M-20 computer for the case of a strong shock wave. The boundary conditions in this case have the form

$$\zeta = 1, \quad \chi = \frac{\varkappa + 1}{\varkappa - 1}, \quad z = \frac{2(\varkappa - 1)}{(\varkappa + 1)^2}$$

$$h = 1 \text{ for } w = \frac{2}{\varkappa + 1}, \qquad y = 0 \text{ for } w = 1$$

The latter no longer depend on ξ_0, and therefore, to solve the problem, one may first set the value of ξ_s (position of the piston) and then determine the parameter ξ_0 (position of the shock wave) from (2.6).

The figure shows plots of $\theta_1(\zeta) / \theta_1(1)$ и $\chi_1(\zeta) / \chi_1(1)$ vs ζ for a gas moving in a magnetic field (solid lines) and in the absence of a magnetic field (dashed line) for $\alpha = 0$, $\varkappa = 5/3$, $n = 3/2$, $\xi_s = 1$, $B_1 = 1$ $(D_1 = 0.44)$, $A_1 = 5 (R_m = 0.3)$ and $A_1 = 16.6 (R_m \approx 1)$. Here, it is kept in mind that for a strong shock wave

$$R_m = \frac{2^{n+1}(\varkappa - 1)^n}{(\varkappa + 1)^{2n+1}} A_1, \quad D_1 = \frac{\varkappa^2 - 1}{4} B_1$$

It is noteworthy that the presence of a magnetic field leads to a deceleration of the shock wave and to a decrease in the gas velocity as compared to the motion of a gas in the absence of a magnetic field. The value of the deceleration increases with increasing interaction parameter $S = D_1 \cdot R_m$. Because of the finite conductivity of the gas, there develops Joule dissipation in the medium, on account of which heat is supplied to the particles of the medium.

It should be noted that the class of similar solutions obtained also permits the formulation of the problem of the motion of a conducting gas in a magnetic field in the presence of instantaneous energy release in the center of symmetry. To this end, it is necessary to set $m = -\frac{1}{2}(1 + \alpha)$. A similar problem was examined in [2] for the case of a strong shock wave.

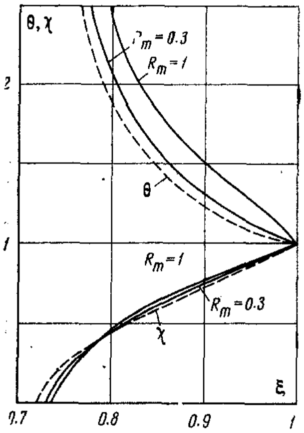
In conclusion, we present another class of similar solutions characterized by an exponential time dependence (in connection with problems in gas dynamics [3, 4]). It is assumed that the gas conductivity depends both on temperature and density, and that it is defined by the formula $\sigma = CT^n\rho^l$, where n and l are certain numbers. Then the similar solution of the system (0.1) has the form

$$H = H_0 e^{\frac{1}{2}(m+3)/kt}\, h(\xi), \quad v = v_0 e^{kt} u(\xi)$$

$$T = \frac{v_0^2}{R} e^{2kt}\theta(\xi), \quad \rho = \frac{P_0}{v_0^2} e^{(m+1)kt}\chi(\xi)$$

$$P = P_0 e^{(m+3)kt} G(\xi)$$

where, as above, m is an arbitrary dimensionless parameter, and k a dimensional constant with the dimensionality $\sec^{-1}$. The similarity variable is $\xi = k/v_0 r e^{-kt}$, H_0, v_0, and P_0 are the corresponding dimensional constants.



The dimensionless functions h(ξ), u(ξ), and so on, are defined by a system of ordinary differential equations

$$[(u-\xi)h' + \left(\frac{m+3}{2} + \frac{\alpha}{\xi}u + u'\right)h = \frac{1}{A}\frac{1}{\xi^{\alpha}}\left(\frac{\xi^{\alpha}}{\theta^{n}\rho^{l}}h'\right)'$$

$$(u-\xi)u' + u = -\frac{1}{\chi}(\chi\theta + Bh^2)', \qquad (u-\xi)\chi' + \left(m+1+\frac{\alpha}{\xi}u + u'\right)\chi = 0$$

$$(u-\xi)\theta' + 2\theta = -\frac{\varkappa-1}{\xi^{\alpha}}\theta(\xi^{\alpha}u)' + \frac{2B(\varkappa-1)}{A}\frac{h'^2}{\theta^{n}\chi^{l}}$$

$$(A = 4\pi C v_0^{2(n+1-l)} P_0^{l} / kR^{n}, \quad B = H_0^2 / 8\pi P_0)$$

From the similarity conditions for the constants n, l, and m, we have the following relation: $2+2n+l(m+1)=0$.

Here, it is necessary to require that $l \neq 0$, $m \neq -1$, and $n \geq 0$.

As in the case of a power-law similarity, examined above, the class of exponential similar solutions we have given permits formulation of the problem of the motion of a conducting gas in a magnetic field under the action of a piston that moves according to an exponential law, while for $m=-4-\alpha$, it permits formulation of the problem of the motion of a conducting medium in the presence of an instantaneous energy release in the center of symmetry.

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LITERATURE CITED

1. G. Birkhoff, Hydrodynamics [Russian translation], Izd-vo inostr. lit., 1963.
2. V. P. Korobeinikov and E. V. Ryazanov, "Influence of a magnetic field on the propagation of plane-symmetrical and axially symmetrical shock waves," PMTF, No. 4, 1962.
3. K. P. Stanyukovich, Unsteady Motions of a Continuous Medium [in Russian], Gostekhizdat, Moscow, 1955.
4. O. Yu. Polyanskii and B. G. Lebedeva, "A class of self-similar motions of a relaxing gas," Zh. vychisl. matem. i matem. fiz., No. 6, 1966.